Ex. Find the minimum point DFT for the following sequence using DIT-FFT algorithm. March 18.

$$x[n] = \{ \underset{\uparrow}{1}, 0, 1, 2, 0, 2 \}$$

$x[n] \rightarrow X[k]$: DIT
$x[n] \leftarrow X[k]$: IIF

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_8^1 = e^{-j\,2\pi/8} = e^{-j\,\pi/4}$$

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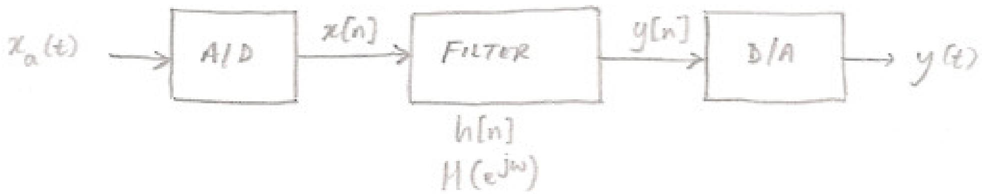
4. DIGITAL FILTER DESIGN TECHNIQUES.

A frequency-selective filter is a system that passes certain frequency components and rejects all others.
Digital filters are implemented by digital computations to filter the signal, which is already converted to digital form



Filter design involves the following steps
(a) specification of desired system properties
*** (b) approximation of the specs. using a causal discrete time system
(c) realisation (implementation) of the system

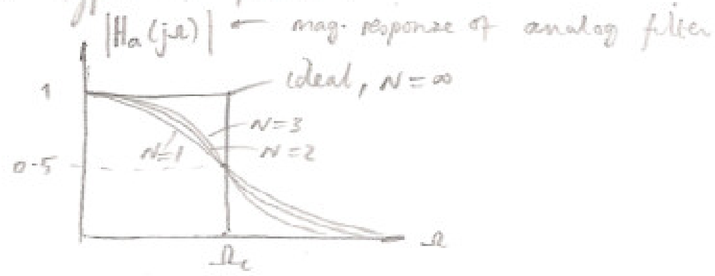
(a) → related to application
(c) → related to device or technology to be used for implementation

Review of Analog Filters.

Within the passband there might be some variation in response. Depending on that, there are two types of filters.

(i) Butterworth

-- N^{th} order of the filter



Consider the T/F of an N^{th}-order analog filter in Laplace domain:

$$\boxed{H_a(s) = \sum_{k=1}^{N} \frac{A_k}{s-s_k} = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \frac{A_3}{s-s_3} + \ldots}$$

$$\therefore h_a(t) = \mathcal{L}^{-1}\{H_a(s)\} = \sum_{k=1}^{N} A_k e^{s_k t}, \quad t \geq 0$$ ← make causal (an approximation)

$$h[n] = h_a(nT_s) = \sum_{k=1}^{N} A_k e^{s_k n T_s} \cdot u[n]$$

$$= \sum_{k=1}^{N} A_k \left(e^{s_k T_s}\right)^n \cdot u[n]$$

$$\therefore \boxed{H(z) = Z\{h[n]\} = \sum_{k=1}^{N} \frac{A_k}{1 - e^{s_k T_s} z^{-1}}}$$

$H(z) = \frac{Y(z)}{X(z)}$ ← now get the difference equation.

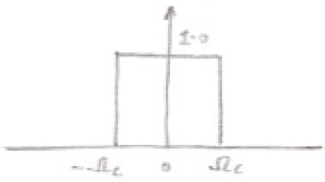
In s-domain, poles are $s = s_k$
In z-domain, " " $z = e^{s_k T_s}$

ADVANTAGE: Relation between ω & Ω is linear ($\omega = \Omega T_s$)

DISADVANTAGE: Aliasing may occur even if Nyquist criteria is valid

We know,

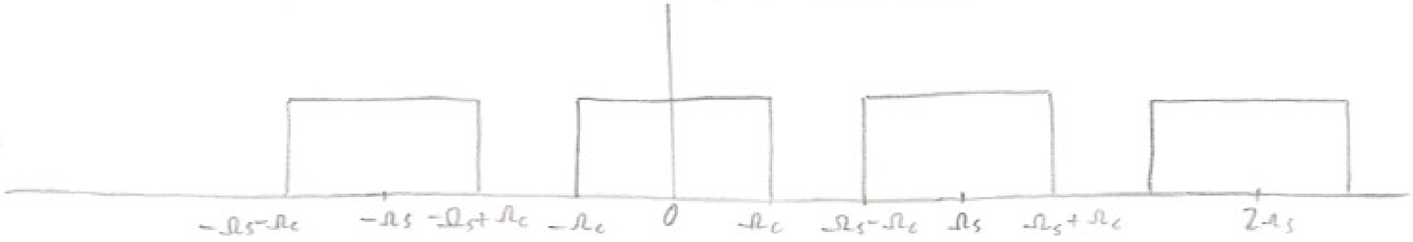
$$H(e^{j\omega}) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} H_a(j\Omega - jk\Omega_s) \Big|_{\Omega = \frac{\omega}{T_s}}$$



For a band limited ideal L.P.F,

$$H_a(j\omega) = 0, \quad |\Omega| > |\Omega_c| \quad \text{where} \quad \Omega_c < \frac{\pi}{T_s}$$

$H(e^{j\omega})$, $\Omega_s > 2\Omega_c$



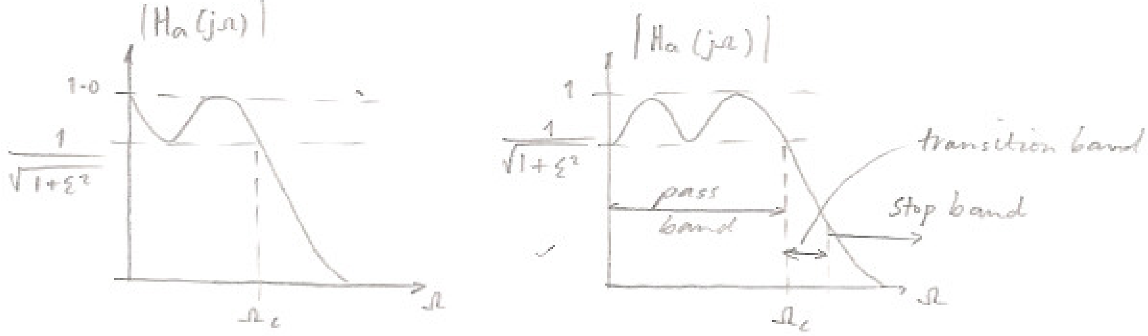
For the ideal case, there is no overlapping. However, for practical analog filters it cannot be exactly band-limited and interference of successive terms ...

For an N^{th} order L.P Butterworth filter,

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}} = H_a(s)\, H_a(-s),$$

where $H_a(s)$: transfer function of the filter in Laplace domain

(b) Chebyshev Filter



N: odd $\qquad\qquad$ N: even

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 T_N^{\,2}\left(\frac{\Omega}{\Omega_c}\right)}$$

T_N : N^{th}-order Chebyshev polynomial

$$T_N(x) = \cos\left(N \cos^{-1} x\right), \quad 0 \leq x \leq 1, \quad x = \frac{\Omega}{\Omega_c}$$
$$= \cosh\left(\cosh^{-1}(x)\right), \quad x > 1$$

§ 4.1 Design of Digital (Discrete-Time) IIR filter from an available analog filter

infinite impulse response

Two methods → ① impulse invariance method (IIM)
→ ② bilinear transformation method (BTM)

① IIM.